

# High frequency QPOs: nonlinear oscillations in strong gravity

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**Abstract.** Some of the more promising ideas about the origin of the high frequency variability (kHz QPOs) in the observed X-ray emissions of low-mass X-ray binaries are contrasted with less promising ones.

**Key words:** general relativity – hydrodynamics – X-ray binaries

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## 1. Setting the stage

### 1.1. Remembering the key crossroads

If I am personally convinced that the high(est)-frequency QPOs in black hole and neutron-star sources have a common origin, it is not only because this assumption has served us well in the past five years, but also because I have taken the other road, and I have seen that it leads nowhere. I would like to begin by describing my journey, so that others may decide whether or not the direction we now take is reasonable.

### 1.2. The accretion-gap paradigm for neutron stars

The discovery twenty years ago of the first millisecond (radio) pulsar immediately brought home the idea that low-mass X-ray binaries (LMXBs) may contain rapidly rotating neutron stars endowed with a magnetic field far below the theretofore canonical value of  $10^{12}$  G. These were hard-to-imagine times when the timing capability of the the best X-ray instrument (GINGA) was below  $\sim 200$  Hz, and (EXOSAT) spectra were only reported up to 30 keV.

In our first paper on LMXBs, formally devoted to investigating spin-up by accretion, we examined the general-relativistic orbits around neutron stars with dynamically unimportant magnetic fields. We have pointed out that for many viable equations of state (of matter at supranuclear densities) the accretion disk could terminate close to the marginally stable orbit (ISCO) well outside the stellar surface (Kluźniak and Wagoner 1985) and dubbed this situation the “accretion-gap regime.” In other words, the accretion disk

of a neutron star in a typical LMXB could well be very similar to the accretion disk around a black hole, with the inner edge determined by GR effects of strong gravity (with frame dragging and all).

The next paper in the short series, prepared as part of my thesis work (Kluźniak 1987), demonstrated that the spectrum of the emission expected in the accretion-gap regime, when the freely falling matter which left the inner edge of the disk hits the equatorial accretion belt, is a rather hard power-law extending up to  $\sim 10^2$  keV (Kluźniak and Wilson 1991). The discovery several years later of such emissions from some neutron star sources can be taken as possibly confirming the presence of the ISCO. However, non-thermal spectra do not uniquely point to the geometry or process of emission, and so we also searched for timing signatures of the accretion-gap regime or, as it is put today, of a key prediction of Einstein’s theory of gravitation (GR).

We were looking not for a tiny quantitative departure from Newtonian gravity, but for a major difference. The time of flight of matter through the gap would be one such signature of a qualitative departure from Newtonian accretion geometry. Accordingly, we computed the trajectory and the fate of a clump of matter orbiting in the accretion disk, crossing the sonic point within the inner edge, and hitting the stellar surface (Kluźniak, Michelson and Wagoner 1990). We proposed this as a method of measuring the mass of the neutron star, or (if the mass were known) of testing GR. In particular, interpreting the 0.2 Hz QPO observed in a certain X-ray pulsar (Angelini, Stella and Parmar 1989) as (an unknown process) occurring at the orbital frequency close to the inner boundary of the accretion disk, which is  $\sim 10^3$  stellar radii above the surface of this strongly magnetized accreting neu-

tron star, we pointed out that in a LMXB neutron star in the accretion gap regime the same QPO should occur at a frequency which a) is in the kHz range, b) inversely depends on the mass  $M$  of the neutron star, and c) depends on the angular momentum  $j$  of the neutron star—the frequency at the ISCO being  $2.2 \text{ kHz } (M_{\odot}/M)(1 + 0.749j)$ . It is in the context of the accretion-gap paradigm that our efforts to understand kHz QPOs should be understood.

### 1.3. Two kHz QPOs in neutron stars, one in black holes?

The discovery in neutron stars of two kHz quasi-periodicities in observations of neutron stars by the first instrument (RXTE) that had the capability of observing such rapid variability did not seem very surprising in light of the Kluźniak, Michelson and Wagoner 1990 prediction discussed above. Neither did the lack of a coherent pulsation associated with the spin of the neutron star—after all, to obtain high orbital frequencies, we had to assume that the stellar magnetic field was too weak to channel the flow. The frequencies came out just right, if the neutron stars were a bit on the heavy side of the allowed mass range in an LMXB transferring mass for  $\sim 10^8 \text{ y}$ .

Soon a high frequency QPO was reported also from a black hole source. This all seemed to make perfect sense. Black hole accretion disks were just like neutron star accretion disks, the only difference being in the size of their inner gap (proportional to mass). Hence, one QPO at the inner edge of the disk was expected, probably associated with an orbiting clump. On top of it, matter falling onto the equatorial accretion belt differentially slipping on the neutron star surface—i.e., rotating faster than the neutron star, albeit at variable speeds—could support a structure (perhaps a vortex like the great red spot on Jupiter, although the referee prevented this specific suggestion from appearing in print) giving rise to the second quasi-period. No such second QPO was expected from a black hole, where matter fell down the horizon. At any rate, identifying the observed frequency with that in the ISCO one could determine the mass of the neutron star or black hole (Kluźniak 1998), a favorite pastime for many authors in the subsequent years.

A model very similar to this (minus the strong gravity part), had been proposed much earlier by Paczyński to explain the white dwarf QPOs, and their variable frequencies, so it all made perfect sense in the kHz QPO context, where both frequencies were known to increase with the luminosity of the source—the idea in the model being that at higher accretion rates the sonic point moves in closer to the stellar surface, and that the top of the equatorial accretion belt moves faster as the accretion rate of angular momentum increases (see Kluźniak, Michelson and Wagoner 1990 for references on the position of the sonic point).

However, when I tried to make the model of the frequency variation in kHz QPOs agree quantitatively with the data, I failed. And I could make no sense of any other published model based on the assumption that the presence of the second kHz QPO is a direct signature of the (spinning) stellar surface. A timely warning from Dimitrios Psaltis that the final

word on the number of QPOs in black holes has not yet been said, as they are very weak at high frequencies, has helped me to finally abandon the idea that only one of the high frequency QPOs arises in the accretion disk. If one arises in the accretion disk, why not both?

### 1.4. Wrong turn

Much initial work on neutron-star QPOs was informed by earlier models of similar phenomena in white dwarf systems (CVs), but the assumption that the kHz QPOs were caused by orbiting clumps (inhomogeneities in the accretion flow) turned out to be wrong. It had been known that clumps in the differentially rotating accretion disk would be sheared out into nearly axisymmetric structures within a few orbits (Bath, Evans and Papaloizou 1974)—i.e., a clump is short lived—and it has simply been overlooked that the kHz QPOs were reported to be very narrow features already in the discovery papers. Today we know that the high quality factor,  $Q \sim 200$  of the lower kHz QPOs in neutron-stars excludes all clump-based models, as well as those involving longer-lived vortices drifting radially inwards with the mean accretion flow (Barret et al. 2005). Thus, the question of relative merits of the “beat frequency model” and the “relativistic precession model” is a moot point.

### 1.5. Linear diskoseismology

All the while, fundamental work was being done on the normal modes of accretion disks. It turns out, that in GR  $g$ -modes are trapped near the maximum of the radial epicyclic frequency (Okazaki, Kato, Fukue 1987), an effect clearly absent in Newtonian  $1/r$  gravity that requires the epicyclic frequencies to be equal to the Keplerian orbital value  $\sqrt{GM/r^3}$ , itself a monotonic function of the radius. After nearly two decades of concerted effort (Wagoner 1999, Kato 2000) several modes have been identified, which were promising in the context of black hole HFPOs. Because this is such a difficult problem all work had been carried out in the linear regime.

One conclusion, more clearly seen *a posteriori*, is that no special ratio of frequencies for any of the preferred low-lying modes is predicted. More precisely, over and above the  $1/M$  scaling in GR, the frequencies of the various modes are a function of the spin of the central compact object, and so is the ratio of frequencies, at least for modes which are not harmonics of the same fundamental. Thus, judiciously choosing the black hole spin one can accommodate nearly any frequency ratio of, e.g., the fundamental  $g$ -mode and  $c$ -mode. However, no two black holes (in a limited sample) are expected to have the same spin, hence any such ratio must be accidental and limited to one or two objects at most.

Another conclusion was that the frequencies have only a slight dependence on the expected variations in the disk structure. Therefore, disk modes were not thought to be directly responsible for modifying the X-ray flux in neutron-star systems, where the QPO frequencies vary quite a bit in any given source.

These two conclusions need no longer hold in non-linear diskoseismology. The equations of hydrodynamics are intrinsically non-linear and there is urgent need to carry out an analysis of disk modes in the non-linear regime. We have reasons to believe that the properties of the high frequency QPOs in both black hole and neutron-star systems are a manifestation of non-linear coupling between certain oscillations of the accretion disk.

## 2. Two characteristic variable frequencies

The two kHz QPOs in neutron stars have reproducible properties in each source, including characteristic frequency values, and yet the two frequencies are not fixed—they vary. It is easy enough for the accreting fluid to display characteristic frequencies. Even in Newtonian gravity, where the orbital frequency is scale-free, there is a maximum frequency corresponding to that in the inner disk—simply because there is a characteristic radius, for example the stellar radius. In GR orbital motion characteristic frequencies are built in, for example the maximum radial epicyclic frequency, and the frequencies of the trapped modes are a reflection of this property of strong gravity. However, all these characteristic frequencies are fixed for a given space-time metric (i.e., a given neutron star or black hole).

It is the variation of frequencies that is highly suggestive of non-linear dynamics. Frequencies that vary in a characteristic range are a well studied feature of non-linear oscillators. For these reason, we have suggested that the two kHz QPOs in neutron stars are a manifestation of a nonlinear resonance in an accretion disk placed in strong gravity (Kluźniak and Abramowicz 2001a,b). Two corollaries immediately follow, and within a few months both were directly verified by observers unaware of our hypothesis. The first is that there are preferred rational ratios of frequencies, such as 1:2 or 2:3. The second is that high frequency QPOs in black holes also come in pairs.

This triumph of simple physics invites more work to be done. In particular, it would be good to identify the two non-linear oscillators, and it is necessary to convincingly identify the similarities and the differences of black hole and neutron-star QPOs. This is the subject of the current workshop.

## 3. The smoking gun of non-linear oscillations

The twin high-frequency QPOs need not have identically the same properties in black holes and neutron-stars. The differences, if any, would presumably be linked to the presence of a rotating magnetosphere in accreting neutron stars, which could directly imprint the spin frequency on the oscillations of the disk.

The stellar spin frequency  $\nu_*$  may couple to another periodic motion already present in the system in more than one way, easily leading to the appearance of a beat frequency  $\nu_2 = \nu_1 \pm \nu_*$ . However, if non-linearities are present, one-half of the spin frequency may appear in the data. We are

fortunate in that nature has provided us with transient systems where the accreting neutron star displays a coherent frequency, surely equal to its spin frequency. In the first of these systems to be closely studied, SAX J1808, two kHz QPOs have been revealed, whose frequencies satisfy  $\nu_2 - \nu_1 = \nu_*/2$ . In our view, this is a clear signature of non-linearities in the system (Kluźniak et al. 2004).

We may even try to give a specific model for the non-linear interaction. Elsewhere, we have shown that certain fluid configurations in the accretion flow, e.g. tori, can execute two quasi-rigid oscillatory motions accurately described by equations nearly identical to that of a test particle in a circular orbit, with the mode frequencies equal to the epicyclic frequencies in a certain orbit (Kluźniak and Abramowicz 2002, Lee et al. 2004). Let us focus on one of these oscillations, with the radial epicyclic frequency  $\omega_r$ . In the presence of the neutron star driver, it could be described by the equation of a non-linear forced oscillator, with a forcing function  $f(t)$  that is periodic at the spin frequency,  $\nu_*$ :

$$\ddot{r} + \omega_r^2 r + a(z)r^2 = f(t). \quad (1)$$

The configuration is also executing a vertical motion, whereby the value of the vertical co-ordinate  $z$  in this equation oscillates at the frequency  $\omega_z$ . We identify the observed frequencies with the frequencies of the oscillators:  $\nu_1 = \omega_r/(2\pi)$  and  $\nu_2 = \omega_z/(2\pi)$ . Reflection symmetry in the equatorial plane requires the expansion of the  $a(z)$  coefficient to only have even powers in  $z$ . The leading coupling term, then, has the form  $a_2 z^2 r^2$ , with  $a_2$  a constant. Substituting the harmonic time dependence of  $z$ ,  $r$ , and  $f$  at frequencies respectively,  $\nu_2$ ,  $\nu_1$ , and  $\nu_*$ , we see that the condition for forced resonance is  $2(\nu_2 \pm \nu_1) = \nu_*$ . For realistic models of the spinning neutron star in J1808, the option  $\nu_2 - \nu_1 = \nu_*/2$  is viable. Some other possible forms of the non-linear equation also yield the same result.

## 4. A model for the 3:2 frequency ratio

The same oscillation model, even without the forcing term and the  $r^2$  non-linearity, allows a simple explanation for the 3:2 frequency ratio that is observed in the high frequency QPOs in black holes, as was noted in Abramowicz and Kluźniak 2001, Remillard et al. 2002, and as we argue, is also present in neutron stars. The epicyclic motions of the fluid configurations, e.g., of a torus are given by the equations

$$\ddot{r} + \omega_r^2 r = 0, \quad (2)$$

$$\ddot{z} + \omega_z(r)^2 z = 0. \quad (3)$$

In view of reflection symmetry in the  $z = 0$  plane, we have neglected the  $z$  variation of the radial epicyclic eigenfrequency. If only one oscillation is present, the frequencies are taken at a fixed equilibrium point, so that the solution is a simple harmonic oscillator motion. Now imagine that both oscillations are present. The radial motion induces harmonic variations of the eigenfrequency  $\omega_z$ , occurring at the frequency  $\omega_r$ , and the vertical motions can be resonantly excited by the horizontal oscillations. This is a situation very

familiar to any child exciting a swing, formally the equation is known as the Mathieu equation, and the resonance (known as the parametric resonance) occurs when

$$\omega_r = \frac{2}{n} \omega_z, \quad (4)$$

with  $n = 1, 2, 3 \dots$  an integer. In general relativity, the first and strongest resonance occurs for  $n = 3$ , leading to a 3:2 ratio of the eigenfrequencies (Kluźniak and Abramowicz 2002), because, in addition to eq. (4), the relation

$$\omega_r < \omega_z$$

must be satisfied.

## 5. The Bursa plot

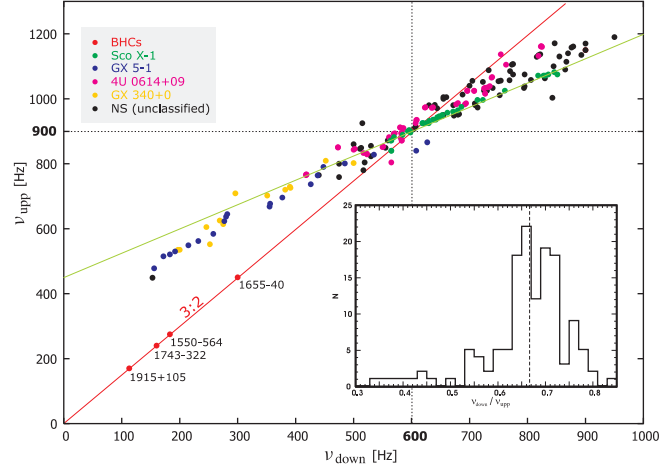
The final question that must be discussed is whether neutron-star QPOs are indeed the same phenomenon as the black hole ones. This is not immediately obvious because the kHz frequencies in neutron stars vary. In fact the frequency ratio of these kHz QPOs departs from 3:2. There are three points to be made.

One is that the histogram of frequency ratios for individual neutron star sources, as well as for all of them jointly, peaks near the value 1.5 (Abramowicz, et al. 2003a). This is apparent in the inset of the figure (where actually the ratio of the lower frequency to the higher is plotted, so that the peak is close to 0.67). The black hole QPOs are much weaker so, at present, we only see the peak of the distribution. It is not known whether in black holes the weaker QPOs, presumably also present, have the same frequency, or whether their frequencies also depart from the 3:2 line.

The second point is that the departure of the observed frequencies from the eigenfrequencies of the system, and of the ratio from the resonant 3:2 ratio, is a generic feature of non-linear resonance in coupled oscillators, so the line of correlation for an individual neutron star can be reproduced in numerical and analytic work (Abramowicz et al. 2003b, Rebusco 2004, Horak 2004).

The third point, as is very apparent in the figure, is that the data points for at least some neutron stars seem to lie along straight lines on the frequency-frequency plot, with a common intersection point. This intersection point lies on the 3:2 line and corresponds to frequencies which are higher than the black hole frequencies. This can be explained by assuming that these neutron stars have very similar masses, smaller than the (more varied) masses of the black holes. The latter already show a  $1/M$  frequency dependence discovered by Remillard and McClintock. The common intersection point of (some of) the neutron star lines greatly strengthens the evidence for the 3:2 resonance in neutron star QPOs, as well as for the strong gravitational field origin of the high frequency QPO phenomenon.

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**Fig. 1.** The Bursa plot. The highest QPO frequency is plotted against its lower frequency counterpart. The black hole frequencies are from Remillard and McClintock 2005. The neutron star twin kHz QPO frequencies are based on published data (see Abramowicz et al. 2003a for references). Note that the neutron star points are scattered about a line intersecting the line of a 3:2 ratio in a single point  $\sim (600 \text{ Hz}, 900 \text{ Hz})$ , and that all black hole frequencies are lower, consistent with a  $1/M$  dependence. The green line going through the Sco X-1 data has a slope of 3/4. This suggests a simple non-linear physical mechanism for its origin.

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